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JBF-003-1161003 Seat No. ________

M. Sc. (Sem. I) Examination

December – 2019

Mathematics : CMT-1003

(Topology - I)

Faculty Code : 003
Subject Code : 1161003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) There are five questions.
(2) Attempt all the questions.
(3) Each question carries equal marks.

1 Answer any seven questions. $7 \times 2 = 14$

a) Define: Closed set. Give an example to show that arbitrary union of closed set need not be closed.

b) Prove that a space (X, τ) is a discrete space if and only if $\forall x \epsilon X, \{x\} \epsilon \tau$.

c) Define: Convergence sequence in a metric space.

d) State Hausdorff's Criterian.

e) Define: Interior of a set. If $A \subset B$ then prove that $A^\circ \subset B^\circ$.

f) Define: Continuity of a function at a point.

g) Prove that locally connectedness is topological property.

h) Define: Co-finite topology.

i) Define: Homeomorphism with an example.

j) Define: Locally path connected space.

2 Answer any two. $2 \times 7 = 14$

a) Prove that lower limit topology on $\mathbb{R}$ is finer than the standard topology on $\mathbb{R}$.

b) Prove that $\tau = \{U \subseteq \mathbb{R}$; for each $x \in U$, there is an open interval $(a, b) \ni (a, b) \subset U\}$

c) Let (X, τ) be topological space. Then prove that

1) $X, \emptyset$ are closed set.
2) Arbitrary intersection of closed set is closed.
3) Finite union of closed set is closed.

3 Answer the following. **$2 \times 7 = 14$**

a) Let (X, τ) be topological space and Y be non-empty subset of X.
Let $\tau_Y = \{G \cap Y;\ G \in \tau\}$.

b) Let X and Y be topological spaces. Then prove that
$\mathcal{B}_{X \times Y} = \{U \times V; U$ is open in X and V is open in $Y\}$ is a basis for some topology on $X \times Y$.

OR

a) If (X, d) be a metric space and $\mathcal{B} = \{Bd(x, \varepsilon)/x \epsilon X, \varepsilon > 0\}$ then prove that $\mathcal{B}$ is a basis for some topology on X.

b) Let X and Y be spaces. $A \subset X$ and $B \subset Y$. Then prove that $\overline{(A \times B)} = \bar{A} \times \bar{B}$

4 Answer any two. **$2 \times 7 = 14$**

a) Suppose X and Y are topological space and $f: X \to Y$ be any function. Prove that f is continuous iff f is continuous at every point of X.

b) State and prove Pasting Lemma.

c) Prove that

1) If $A \subset X$ then $\overline{A} = \{x \in X$, for any open set U containing $x, U \cap A \neq \emptyset\}$.
2) If $A \subset X$ then $\overline{A} = A' \cup A$.

5 Answer any two. **$2 \times 7 = 14$**

a) Prove that $X \times Y$ is a locally path connected if and only if X and Y are locally path connected.

b) If X is connected and locally path connected then prove that X is path connected

c) Suppose X and Y are topological space. If $f: X \to Y$ is continuous and onto. If X is connected then prove that Y is also connected

d) Prove that

1) If C is a component and A is a connected set then either $A \cap C = \emptyset$ or $A \subset C$.
2) If C is a component then C is a maximal connected subset of X.
3) If C is a component then C is a closed subset of X.